1-cap Planarity

Abstract

(Joint work with C. Baer, O. Genty, T.W. Mattman, R. Naimi, A. Pavelescu, and G. Wahl)

Planar graphs are graphs that can be embedded in the plane in such a way that no edges intersect each other. In this talk, we introduce the idea of 1-cap planarity. A *cap* is defined to be either the edges of a 3-cycle or a vertex. A graph is 1-cap planar if it is planar or can be made planar by removing at most 1 cap. We show that 1-cap planar graphs have a *linkless embedding.* This means the graph can be embedded in the 3-sphere in such a way that there are no non-trivial links present. Similarly, a graph has a *knotless embedding* if it can be embedded in the 3-sphere without non-trivial knotted cycles.

We say a graph G is an *obstruction* for some property P if G does not have property P and every proper minor $H \lneqq G$ does have that property. A *minor* of a graph is a topological subgraph. The Peterson family of 7 graphs are the obstructions to linkless embeddings. Finding the obstructions for knotless embeddings is an interesting open problem in knot theory and, while we understand the obstructions for linkless embeddings well, understanding the obstructions for knotless embeddings is difficult. We explain how progress in identifying obstructions for 1-cap planar graphs advances our comprehension of linkless and knotless embeddings.